Subradiance localization in the open 3D Anderson-Dicke model

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Anderson localization is a paradigmatic coherence effect in disordered systems, often analyzed in the absence of dissipation. Here we consider the case of coherent dissipation, occuring for open system with coupling to a common decay channel. This dissipation induces cooperative Dicke super- and subradiance and an effective long range coupling, expected to destroy Anderson localization. We are thus in presence of two competing effects, i.e localization driven by disorder and delocalization driven by dissipative opening. Here we show that in an open 3D Anderson model, subradiance enables the system to preserve signatures of a localization transition. We demonstrate the existence of a *subradiant localized regime*, emerging from the interplay of opening and disorder, in which subradiant states are hybrid and localized, while superradiant states are extended. We also provide analytical predictions for this regime, confirmed by numerical simulations.

One of the most important effect induced by quantum coherence is Anderson localization [1]. This effect is relevant for many areas of Physics, quantum computing [2, 3], cold atoms [4], light harvesting systems [5], mesoscopic electronic devices [6]. Initial search of experimental evidence of Anderson localization for non interacting waves in 3 dimensions has been limited by the presence of absorption [7]. Recent progress has however been obtained in a large variety of fields of research, including acustic waves [8], light waves [9], matter waves [10].

Whereas absorption and dissipation is often considered to be limiting phenomena for observing Anderson localization, the situation of opening with corresponding coherent dissipation has not been systematically addressed. This situation has been considered in a different research community [5, 11], investing cooperative effect induced by coupling to a commun decay channel, following the pioneering work by Dicke in 1954 [12]. The question whether localization can survive coherent dissipation in open 3D system has been discussed in particular in the case of resonant light scattering [4, 13].

In this letter we focus on the case of coherent dissipation due to the coupling to a common decay channel, relevant for many realistic situations [3, 4, 14]. If this coupling is strong enough, this kind of dissipation induces a well known important coherence effect: Dicke super- and subradiance [12], which induces the so-called Superradiance Transition (ST), driven by the opening, i.e. by the coupling to an external environment characterized by a continuum of states. This transition is to be compared on the other hand to the Anderson localization, driven by intrinsic local disorder which consists in the suppression of diffusion due to exponential localization of all the eigenfunctions of the system.

Both Dicke superradiance and Anderson localization have been widely studied in literature in a separate way, and their interplay has been poorly analyzed. In order to highlight the fundamental aspects of this interplay, we have choosen to analyze a simple but general model: the 3D Anderson model [1]. A related 1D model, without a metal-insulator transition, has been already considered in Ref. [15]. A common feature of disordered system with coherent dissipation is the competition between long range hopping induced by opening and localization induced by disorder. It is commonly accepted that any kind of long range hopping, decaying slower than $1/r^d$, where d is the system dimension, destroys localization [1]. Here we show on contrary that the very correlated nature of long range hopping due to the opening induces a Superradiant Transition which allows to preserve localization.

Dicke superradiance occurs in the large opening regime and it cannot be treated by perturbation theory. In such a case the effective non-Hermitian Hamiltonian approach to open quantum systems has been shown to be very effective [16]. Non-Hermitian Hamiltonians have been already employed in random matrix theory [17–20], in paradigmatic models of coherent quantum transport [21, 22] and in realistic open quantum systems [4, 14, 23–25].

In the closed 3D Anderson model, a particle hops between neighbors sites of a 3D cubic lattice with N sites, in the presence of on–site disorder. The Hamiltonian of the closed 3D Anderson model can be written as:

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j \rangle} (|j\rangle\langle i| + |i\rangle\langle j|) , \qquad (1)$$

where the summation $\langle i, j \rangle$ runs over the nearest-neighbor sites, E_j are random variables uniformly distributed in $[-W/2, +W/2]$, W is a disorder parameter, and Ω is the tunneling transition amplitude. The nature of the eigenstates of the 3D Anderson model depends on

the degree of disorder: for small disorder the states in the middle of the energy band are extended, while close to the band edges, for energies below the *mobility edges*, they are localized [26]. On increasing W/Ω, the mobility edges approach one to each other and above the critical value $W/\Omega \approx 16.5$ [26], all states became localized and a global AT occurs. In the localized regime the shape of eigenfunctions behaves as $|\psi(j)| \sim \exp(-|\vec{x_j} - \vec{x_0}|/\xi)$, where $\vec{x_j}$ are position vectors and ξ is the localization length. In this work we will disregard the effect of the mobility edges, focusing on the global AT:

$$AT \quad \text{at} \quad W/\Omega = 16.5 \qquad (2)$$

Our main question is about the effect of opening on the localization transition which occurs at AT. We open the $3D$ Anderson model by allowing the particle to escape the system from any site into the same continuum channel. This situation of "coherent dissipation", can be met in many realistic systems [3, 4], when the wavelength of the particle in the continuum channel is comparable with the sample size. The case of only one channel in the continuum is somehow extreme, but we expect that our findings have general validity whenever more states compete to decay in the same channel. This kind of opening is different from a standard coupling to a thermal bath (for instance described by time dependent diagonal terms in the Hamiltonian [27]), and takes into account only the particle escape from the system (dissipation). The open system is described by the effective non-Hermitian Hamiltonian [15]:

$$(H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2} \sum_c A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}, \quad (3)$$

where H_0 is the Anderson Hamiltonian, Eq. (1), A_i^c are the transition amplitudes from the discrete states i to the continuum channels c. In our case we have one decay channel, $c = 1$, equal couplings ($A_i^1 = \sqrt{\gamma}$) so that $Q_{ij} = 1 \, \forall i, j$ is a full matrix. The complex eigenvalues of H_{eff} can be written as, $E_r - i\frac{\Gamma_r}{2}$, where Γ_r are the decay widths of the states. Since the average width is given by γ [16], the degree of resonance overlap is determined by $\kappa = \gamma/D$, where D is the mean level spacing of the energy levels of H_0. We can regard κ as an effective degree of opening and at $\kappa \simeq 1$ [16, 18] a segregation occurs, i.e. almost the entire decay width is allocated to just one short-lived "superradiant" state, while all other $N - 1$ states become "subradiant" with a small decay width. We refer to this segregation as the ST. We note that our κ parameter bears some resemblance to a often used Thouless parameter g, which in the case of opening at the edges of the system has been shown to provide relevant information on a metal-insulator transition [28]. Here, the opening is obtained by coupling all sites to a commun decay channel, and our κ thus does not reveal the sensitivity to the boundaries of the system.

The presence of disorder affects ST since D depends on disorder; indeed for small disorder, the width of the energy band is given by Ω and $D \simeq 12\Omega/N$ [1], while for
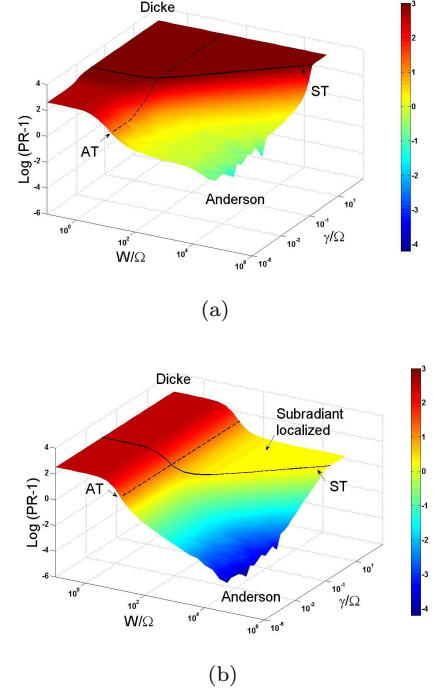

FIG. 1: $Log(PR - 1)$, where PR is the participation ratio, is plotted in the γ/Ω and W/Ω plane. As a dashed line AT is shown, Eq.(2), while the full line refer to the ST at $\kappa = 1$, Eq.(4), so that the region above it $\kappa > 1$ is characterized by superradiant and subradiant states, differently from the region below it ($\kappa < 1$). For an explanation of the different regimes see text. Panel (a) refers to the state with the largest width, which become superradiant above the ST. Panel (b) refers to the other $N-1$ states which become subradiant above the ST. In both panels the system is a cubic lattice with $N = 10 \times 10 \times 10$ sites and $\Omega = 1$. Each data is obtained averaging over 5 realizations of disorder, for panel (b) an additional average over all states has been done.

large disorder, $D \simeq W/N$, so that,

$$ST \quad \text{at} \quad \kappa \simeq \frac{\gamma}{D} = 1 \quad \text{with} \begin{cases} \kappa \simeq \gamma N/12\Omega & \text{if } W/\Omega \ll 12 \\ \kappa \simeq \gamma N/W & \text{if } W/\Omega \gg 12 \end{cases} \qquad (4)$$

So, above ST ($\kappa > 1$), we have a superradiant regime, with superradiant and subradiant states, while below ST($\kappa < 1$), all states are affected by the opening in a similar way. From Eq.(4) it is clear that we can control the effective degree of opening κ by varying the strength of the disorder.

Since Q is a full matrix the opening induces an effective long range hopping among the sites, which is generally expected to destroy localization [29]. The long range hopping can be explained as a second order effect of coupling via a commun vacuum mode which results in the fact that the particle escaping from one site can be reabsorbed in far sites before leaving the system. This interaction mediated by the continuum, is at the origin

of the ST. Thus disorder and opening have opposing effects: while disorder tends to localize the eigenfunctions, the opening tends to delocalize them due to the induced long range hopping.

In order to analyze the interplay of disorder and opening we first study the participation ratio, $PR = \langle [\sum_i |\langle i|\psi\rangle|^4]^{-1} \rangle$ of the eigenstates $|\psi\rangle$ of H_{eff} in Eq. (3), where $\langle \ldots \rangle$ stands for the average over disorder. The PR is widely used to characterize localization properties [30]: for extended states it increases proportionally to the system size, while it is independent for localized states. Note that since $PR \geq 1$ in the following we will analyze $PR - 1$.

This interplay between disorder and opening generates a rich phase diagram, as shown in Fig. 1, where we plot $Log(PR - 1)$ in the γ/Ω–W/Ω plane for a system with $N = 10^3$ sites. In the same figure we also show AT (dashed curve) and ST ($\kappa = 1$ full curve). In the upper panel of Fig. 1 we analyze the state with the largest width which become superradiant above ST, while in the lower panel we considered the other $N - 1$ states, which become subradiant above ST. As one can see the response to disorder in the superradiant regime is very different for superradiant and subradiant states: while the former do not feel AT and remains delocalized up to ST, the latter are sensitive to AT. The reason of this different behavior can be explained from perturbation theory and it is due to the fact that the superradiant state is very far from the complex energies of the other states and thus it is less affected by the disorder [15]. Below the ST, all states feel the disorder and the opening in a similar way. From Fig. 1 it is also clear that the parameters which determine the nature of the eigenstates are W/Ω and the effective coupling strength to the continuum κ, Eq.(4). From Fig. 1 we can distinguish three main regimes:

Region I (at the left of the AT line in Fig. 1): most of the states of the closed system are delocalized, and the opening does not change their extended nature. For this reason we do not distinguish between $\kappa < 1$ and $\kappa > 1$. We neglect the discussion about the effect of the opening on the mobility edges of the closed system, since we are interested in the global transition to localization at AT. Note that for large opening and small disorder we recover the usual Dicke regime.

Region II (right of the AT line and below the ST line in Fig. 1): the states of the closed system are localized and even if the effective opening is small (we do not have superradiance), it induces hybrid states (see discussion below). It is important to note that this region disappears for large N values, since, from Eq. (4) the critical ST line is, $\gamma/\Omega = 12/N$ for $W/\Omega \ll 12$ and $\gamma/\Omega = W/(\Omega N)$ for $W/\Omega \gg 12$. Note that in the limit of large disorder and small opening we recover the Anderson localized regime.

Region III (right of the AT line and above the ST line in Fig. 1): the states of the closed system are localized, but the effective opening is large and we have superradiance. This regime is very interesting since the superradiant state is fully delocalized (Fig. 1, upper panel), while
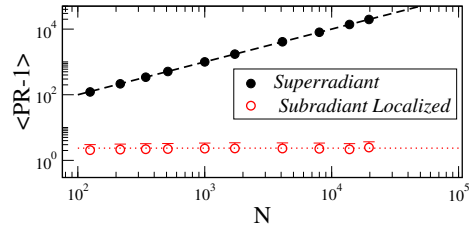

FIG. 2: $3D$ Anderson model. Average participation ratio $\langle PR - 1 \rangle$ *vs* N (up to $N_{MAX} = 27^3$). Here $\gamma/\Omega = 1$ and $W/\Omega = 80$, so that we are in the *Subradiant localized regime*. Full circles stand for the superradiant state, and open red circles stand for subradiant states. Each point has been obtained by averaging both over the states and disorder. Dashed lines are plotted to guide the eye.

the subradiant states are hybrid and localized (see discussion below and Fig. 3): The existence of this regime, which we call the *subradiant localized regime*, is the main result of this work.

A full analysis of the three regions will be presented in a future work. Here we focus on the *subradiant localized regime*, which is the regime with very large opening, in which one might expect no signature of localization.

In order to establish the localized or extended nature of the states in the different regimes, it is necessary to analyze the scaling of the PR with the system size. In Fig. 2, $PR - 1$ is shown *vs* N for the subradiant and the superradiant states in the *subradiant localized regime*, at fixed strength of disorder. As one can see the superradiant states (full circles) are extended, $PR \propto N$, while, surprisingly, we find that the subradiant states behave like localized states with a PR independent of N (open red circles). We have also checked the behavior of the PR in other regions (not shown): Below AT (*Region I*) we find that the states are extended ($PR \propto N$), as in the closed model. In (*Region II*), we have found $PR \propto 1 + N$ which would indicated extended states. In reality the limit $N \to \infty$ can not be performed since at $N \simeq W/\gamma$ we cross the ST, entering the *subradiant localized regime* and the *Region II* disappears. We can conclude that this is a finite-sized effect.

The behavior of the PR is not enough to characterize the nature of the eigenstates ψ, it is also important to analyze their structure, by means of the average density profile, and defined as $\langle |\psi|^2 \rangle$. In the *subradiant localized regime* the superradiant states are fully extended and are well approximated by the following expression: $|Superradiant\rangle \approx \frac{1}{N} \sum_i |i\rangle$, where $|i\rangle$ are the site states of the Anderson model. The analysis of the subradiant states in the *subradiant localized regime* shows that they are hybrid states [14] with an exponentially localized peak plus a constant plateau. The exponentially localized peak behaves like a localized state of the closed Anderson model, compare respectively solid and dashed curves in Fig. 3. On the other hand, the height of the
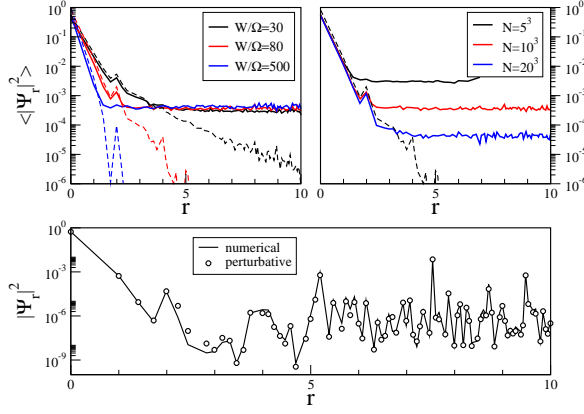
FIG. 4: Average width *vs* W/Ω for $N = 10^3$, and $\gamma = \Omega = 1$. (Black) circles stand for the width of the superradiant state, while (red) crosses stand for the average widths of the $N - 1$ subradiant states. The vertical dashed line represents the AT, while the full vertical line indicates the superradiant transition (ST).

FIG. 3: All data refer to the *subradiant localized regime*. Full curves are for the open system, while the dashed curves are for the closed one. In the two upper panels, the average density profile of subradiant states as a function of the distance r from the heighest Anderson peak is shown. Left upper panel: $N = 10^3$, $\gamma/\Omega = 100$ and different value of disorder as indicated in the legend. Right upper panel: $W/\Omega = 80$, $\gamma/\Omega = 1$ and different system sizes as indicated in the legend. Lower panel: A single hybrid subradiant state is compared with perturbation theory, Eq.(5).

constant plateau is independent of both W and γ, see Fig. 3 (left panel), and it decreases with the system size as $1/N$, Fig. 3 (right panel).

For large disorder, $W \gg \Omega$ and above ST (*subradiant localized regime*), following Ref. [15, 20] it is possible to obtain an analytical expression for the localized subradiant states:

$$|\mu\rangle = \frac{1}{\sqrt{C_\mu}} \sum_{j^0=1}^{N} \frac{1}{\tilde{\epsilon}_\mu - E_{j^0}} |j^0\rangle \text{ with } \sum_{j^0=1}^{N} \frac{1}{\tilde{\epsilon}_\mu - E_{j^0}} = 0. \quad (5)$$

where C_μ is a normalization factor, $|j^0\rangle$, E_{j^0} are the eigenstates and eigenvalues of the closed Anderson model, Eq.(1), and $\tilde{\epsilon}_\mu$ are defined by the constraint given in the second equation of Eq.(5). Note that $\mu = 1, .., N - 1$ spans the subradiant subspace and each $\tilde{\epsilon}_\mu$ lies between two neighbor levels E_{j^0}. Even if Eq.(5) describes very well the localized subradiant states, see Fig. 3 lower panel, it is not trivial to derive a close expression for the density profile.

We have also computed the density profile of the states for small opening and large disorder *Region II*. When the effective opening is small (below ST), the problem can be treated by standard perturbation theory and similarly to what was found in [15], we find that also in this case we have hybrid states, with a plateau height proportional to $(\gamma/W)^2$ but independent of N. The different scaling of the plateau height with the system size, is at the origin of the different scaling of the PR of the open system between *Regions II* and the *subradiant localized regime*. Indeed for very large disorder, the hybrid states in both regions, are mainly localized on only one site with prob-
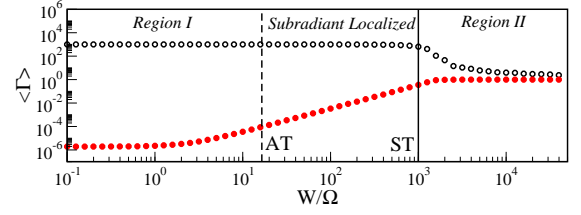
ability a and on all the other $N - 1$ sites with probability b, and we have $\sum_i |\psi_i|^4 = a^2 + b^2(N - 1)$, with $a + b(N - 1) = 1$. So that when $b \propto 1/N$ (*subradiant localized regime*), the PR is independent of N in the large N limit, while, when b is independent of N (*Region II*) one has $PR \sim 1 + 2bN$ in the limit $bN \ll 1$, in agreement with the discussion given above.

A signature of the AT can also be found in the behavior of the decay widths of the subradiant states. In Fig. 4 we show the decay width of both the superradiant and subradiant states as a function of the disorder. While the former is not affected by disorder up to the ST, the latter feels the disorder above the AT (note that, increasing the disorder at fixed γ, the effective degree of opening is decreased and the three regions are crossed). For very large disorder (*Region II*), all widths approach the same value $\Gamma_0 = \gamma$, which corresponds to the decay width of an isolated site.

Conclusions We analyzed coherent effects induced by the interplay of coherent dissipation and disorder. For this purpose we considered a $3D$ open Anderson model in which a particle can escape from any site to a common channel in the continuum. This kind of opening induces a strong long range hopping (all–to–all) between the sites of the Anderson model. Contrary to expectations, we show that the opening does not destroy all features of Anderson localization. Remarkably, for large effective opening, where we have superradiance, we established the existence of a *subradiant localized regime* with extended superradiant states, and localized subradiant states (with a size independent participation ratio). We determined, both numerically and analytically, that the subradiant localized states have an hybrid nature, with an exponentially localized peak and a constant plateau.

This surprising result can be only explained by the highly correlated nature of the long range hopping present in this kind of systems. These correlations induce a Superradiant transition where the coupling with the external world is taken by the superradiant state, leaving the subradiant states effectively decoupled, and thus able to preserve the localized nature of the closed system. Let us observe that recently in Ref. [31], localized states have been found in a related model with hermitian long range

hopping, thus supporting the generality of our results.

Our analysis is relevant both from an applicative point of view, in the search of Anderson localization in $3D$ dissipative systems (i.e. cold atoms), and from a theoretical point of view (role of long range hopping in the metal-insulator transition).

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